# FINANCE WEB APP
# SPREAD OPTION

*Pricing Spread options in the bivariate geometric Brownian motion model*

THIS web app implements Caldana and Fusai (2013) "A general closed-form spread option pricing formula". They propose an accurate method for pricing European spread options in a general model setting. However, the web app implements the pricing formula with reference to the geometric Brownian motion case. In this model setting, the approximation is equivalent to the one proposed by Bjerksund and Stensland (2014).

## Background information

Let $S_1(t)$ and $S_2(t)$ be two stock price processes. An European spread option pays at the maturity date $T$ the amount

$$C_K(T) = (S_1(T) - S_2(T) - K)^+$$

The time 0 no-arbitrage fair price of the spread option is

$$C_K(0) = e^{-rT}\mathbb{E}\left[(S_1(T) - S_2(T) - K)^+\right] \tag{1}$$

where the expectation is with respect to a risk-neutral measure and $r$ is the riskless interest rate. Here, we have used the usual notation $x^+$ for the positive part of $x$, that is, $x^+ = \max\{x, 0\}$. If $K = 0$ and $S_1(t), S_2(t)$ are jointly log-normal, computation of (1) provides the so-called Margrabe (1978) exchange option formula. Very little regarding non-zero strikes and non-Gaussian processes is discussed in the literature, despite the relevance of a closed pricing formula in a number of financial applications. Let us define the event $A$:

$$A = \left\{\omega : \frac{S_1(T)}{S_2^{\alpha}(T)} > \frac{e^k}{\mathbb{E}\left(S_2^{\alpha}(T)\right)}\right\} \tag{2}$$

and let us consider the following lower bound to the spread option payoff:

$$(S_1(T) - S_2(T) - K)^+ \geq (S_1(T) - S_2(T) - K)\, 1_{(A)}$$

The lower bound to $C_K(0)$ (1) is given by:

$$C_K^{k,\alpha}(0) = e^{-rT}\mathbb{E}\left[(S_1(T) - S_2(T) - K)\, 1_{(A)}\right]$$

Indeed it holds that $C_K^{k,\alpha}(0) < C_K(0)$ for all $k$ and $\alpha$. The explicit calculation of $C_K^{k,\alpha}(0)$ is given in Caldana and Fusai (2013) in terms of a Fourier inversion formula as:

$$C_K^{k,\alpha}(0) = \left(\frac{e^{-\delta k - rT}}{\pi}\int_0^{+\infty} e^{-i\gamma k}\Psi_T(\gamma;\delta,\alpha)d\gamma\right)^+ \tag{3}$$

where $\Psi_T(\gamma;\delta,\alpha)$ is given by

$$\frac{e^{i(\gamma-i\delta)\ln(\Phi_T(0,-i\alpha))}}{i(\gamma-i\delta)}\left[\Phi_T((\gamma-i\delta)-i,-\alpha(\gamma-i\delta))\right.$$
$$\left.-\Phi_T(\gamma-i\delta,-\alpha(\gamma-i\delta)-i) - K\Phi_T(\gamma-i\delta,-\alpha(\gamma-i\delta))\right]$$

and

$$\alpha = \frac{F_2(0,T)}{F_2(0,T)+K}, \tag{4}$$

$$k = \ln\left(F_2(0,T)+K\right). \tag{5}$$

The quantity $F_2(0,T) = \mathbb{E}[S_2(T)]$ in formulas (4) and (5) is the forward price of the second asset at time 0 for delivery at future date $T$.

## Form Field

There are 5 different panels (Figure 1) where the user can insert the needed input parameters. Here we illustrate the content of each panel.

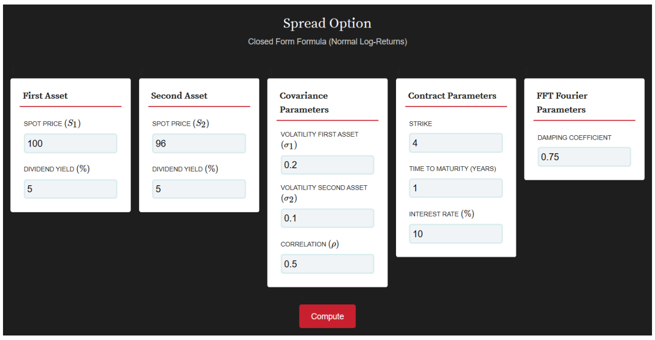


**Figure 1:** Spread option input form

### First asset

The user must insert the relevant quantities for the first stock.

- ❑ Spot price ($S_1$): initial price of underlying asset.
- ❑ Dividend yield (%): annual dividend yield continuously compounded.

### Second asset

The user must insert the relevant quantities for the second stock.

- ❑ Spot price ($S_2$): initial price of underlying asset.
- ❑ Dividend yield (%): annual dividend yield continuously compounded.

### Covariance Parameters

The user must specify the volatility of log-returns of the two stocks and their covariance. All these quantities must be on an annual basis.

- ❑ Volatility first asset ($\sigma_1$): standard deviation of the first asset (annualized).
- ❑ Volatility second asset ($\sigma_2$): standard deviation of the second asset (annualized).
- ❑ Correlation ($\rho$): correlation between $S_1$ and $S_2$ .

### Contract Parameters

The user must specify the characteristics of the spread option contract and the risk free rate (on annual basis and continuously compounded).

- ❑ Strike: strike price.
- ❑ Time to maturity (years): time to expiry (in years) of the Spread option.
- ❑ Interest rate (%): annual risk free rate continuously compounded.

### Fourier parameters

- ❑ Damping coefficient: decay parameter needed to guarantee the existence of the Fourier transform.

## The output

The Table 1 presents the approximated spread option value $C_K^{k,\alpha}(0)$, as given in formula (3). The underlying model for stock prices is the bivariate geometric Brownian motion.

**Result**

Spread Option Price: 6.653058

**Table 1:** Approximated spread option value

### References

Bjerksund, P., & Stensland, G. (2014). Closed form spread option valuation. *Quantitative Finance*, *14*(10), 1785–1794.

Caldana, R., & Fusai, G. (2013). A general closed-form spread option pricing formula. *Journal of Banking & Finance*, *37*(12), 4893–4906.

Margrabe, W. (1978). The value of an option to exchange one asset for another. *The journal of finance*, *33*(1), 177–186.